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ORIGINAL ARTICLE

Artifacts and signs after a Vygotskian perspective: the role of the teacher

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Abstract The notion of mediation, widely used in the current mathematics education literature, has been elaborated into a pedagogical model describing the contribution of integrating tools to the human activity, and to teaching and learning mathematics in particular. Following the seminal idea of Vygotsky, and elaborating on it, we postulate that an artifact can be exploited by the teacher as a tool of semiotic mediation to develop genuine mathematical signs, that are detached from the use of the artifact, but that nevertheless maintain with it a deep semiotic link. The teaching organization proposed in this paper is modeled by what we have called the didactical cycle. Starting from assuming the centrality of semiotic activities, collective mathematical discussion plays a crucial role: during a mathematical discussion the intentional action of the teacher is focused on guiding the process of semiotic mediation leading to the expected evolution of signs. The focus of the paper is on the role of the teacher in the teaching–learning process centered on the use of artifacts and in particular a dynamic geometry environment. Some examples will be discussed, drawn from a long-term teaching experiment, carried out over the past years as part of a National project. The analysis is accomplished through a Vygotskian perspective, and it mainly focuses on the process of semiotic mediation centered on the use of artifacts and on the role of the teacher in this process.

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1 Introduction

The relationship between artifacts and knowledge is complex and asks for a careful analysis in order to avoid useless oversimplification and allow to fully exploit the potential that the use of technology (and in particular of new technology) offers to mathematics education. The issue of integrating technological tools into school practice has become an urgent issue that claims for specific theories, appropriate for the specific situation where the use of tools aims to foster the learning of mathematics. This requires specific paradigms to get insight into the teaching–learning process, to inspire the design of teaching sequences, in short to improve mathematics education. In the past 20 years, a different theoretical framework has become relevant for the issue of integrating technological tools into mathematics education (an overview and a discussion on that can be found in Drijvers, Kieran and Mariotti, forthcoming). This paper intends to contribute to this issue by presenting a paradigm and a specific model emerging from the application of such paradigm in the classroom.

Some theoretical results coming from a number of research projects, some of which carried out by the authors, are fully discussed in (Bartolini Bussi & Mariotti, 2008). Our research projects were carried out at different school levels; however, they shared a few key features like the following. All of the projects were based on the use of (potentially different) artifacts, and on a common methodological frame.

Ever since we started, we found in Vygotsky elements that resonated with our intuitions on the role of artifacts and of signs derived from their use in the construction of knowledge. The key element on which our theoretical model has been developed is that of *semiotic mediation* (Vygotsky, 1978), which sees knowledge-construction as a consequence of instrumented activity where signs emerge and evolve within social interaction. This general framework was enriched and enhanced by a historical and epistemological analysis, and then developed into a pedagogical model. As will be explained in the paper, we identify a connection between the use of particular artifacts and sense-making consistent with mathematical knowledge, and we advance the hypothesis that this connection can be used for didactic purposes.

As far as the common methodology is concerned, according to a long-standing tradition in our country, the main framework has been that of research for innovation (Arzarello & Bartolini Bussi, 1998), where action in the classroom is the goal and the source of theoretical reflections. This appears to be highly consistent with the aims of *research design*, and in particular with the aim that Van den Akker et al. (2006) underline quoting other authors:

> design research aims at *developing empirically grounded theories* through combined study of both the process of learning and the means that support that process (DiSessa & Cobb, 2004; Gravemeijer, 1994, 1998). (Van den Akker et al., 2006, p. 3).

One of the main features of our methodological approach is the development of long-term teaching experiments within the current school activities of regular classes. A basic requirement for such methodology is the continuous collaboration between researchers and teachers, who constitute a team that works together both in the design and in the analysis of teaching sequences, sharing the basic assumptions and discussing step-by-step the consistency between what happens in the class and what is expected from theoretical assumptions. The feedback coming from classroom experience nurtures the development of an original theoretical model as well the didactic sequence that is both a product of the research study and a means to frame further experimentations.

However, within the research team the roles of the different actors with respect to the classroom activities are clearly defined: the teacher has the full responsibility of his/her action in the classroom, the researcher has the role of an external observer.

This collaboration between teachers and researchers was the origin of our reflection upon the teacher's role, based on the designed and observed teacher's action, which became object of investigation in itself.

Besides studies providing information about teachers' knowledge deficiencies in different topics (see Da Ponte & Chapman, 2006 for an overview) or about the relationship between teachers' beliefs/conceptions and teaching (Thompson, 1992; Leder et al., 2002), other studies addressed the role of the teacher and his/her action in teaching–learning activities. These studies were developed from within different theoretical perspectives, most of them sharing the claim that learning takes place in a social setting and stems from interaction. An interactive perspective in teaching and learning has been largely discussed, inside and outside the research field of mathematics education, and different paradigms have been outlined. Some of such paradigms see interaction and learning as participation (Lave & Wenger, 1991), while others focus on collaborative processes, stressing the need for students to work cooperatively, and indicating ways to develop students' abilities to collaborate. The key idea is clearly explained by Bauersfeld:

> teaching and learning mathematics is realized through *human interaction.* It is a kind of mutual influencing, an interdependence of the action of both teacher and student on many levels. […] the student's reconstruction of meaning is a construction via social negotiation about what is meant and about which performance of meaning gets the teacher's (or peer's) sanction. (Bauersfeld, 1980, p. 35).

Our perspective, residing within the stream of social interaction and deeply inspired by a Vygostian approach, claims that a purposeful teacher's action in a social setting is to be considered a key element for students' learning. In the following section a specific model of teacher's intervention will be given. Such model is consistent with the general model describing teaching and learning processes centered on the use of an artifact.

2 Internalization, semiotic processes and the asymmetry of the interlocutors

Vygotsky's approach to learning is not separable from his approach to teaching, and the central role played by internalization constitutes the unifying element (Vygotsky, 1981, p. 162). According to the Vygotskian perspective, internalization takes place in social interchanges inserted into a special register of speech called *discourse*:

> the genre of communication in which the utterances of each interlocutor are *determined by the position they occupy in a certain specific social formation, not just by the speech content to which they refer.* (Carpay & Van Oers, 1999, p. 302).

In a didactic context, as the discourse develops in the classroom, the status of the interlocutors is a-symmetric. The discourse is characterized by simultaneously developing on two different planes: that of the students and that of the culture.

The teacher, as an expert representative of mathematical culture, participates in the collective discourse to help it advance. This help is based on his/her intention inspired by the didactic goal she/he has in mind: for example, evaluation and control of solution-strategies for the activity, or sense-making within mathematics. Success in educational projects is deeply indebted to the teacher's ability to fuel and control the dialectic, following two directions: fostering the evolution of shared meanings, and guiding towards consistency with didactic goals. Specifically, taking into account the cultural perspective ensures consistency and meaningfulness of shared meanings with respect to mathematics as a cultural product and as a teaching and learning objective.

Within this perspective, we carried out our investigation focusing our analysis on the teacher and, specifically, on the teacher's contribution to the development of a mathematical discourse in the classroom, in the specific case of school activities centred on the use of an artifact.

3 A teaching–learning model

In order to describe the teacher's intervention in the education process, we need to give a short account of the teaching–learning model within which the teacher's intervention is conceived. This model constitutes the basic frame within which the specific teaching sequences carried out during our research team were both designed and analysed.

3.1 The semiotic potential of an artifact

The model that we elaborated is based on the seminal idea of semiotic mediation introduced by Vygotsky (1978) and it aims to describe and explain the process that starts with the students' use of an artifact to accomplish a task and leads to the students' appropriation of a particular mathematical content. The learning process, centered on the use of an artifact, is often expressed in terms of mediation (Meira, 1998, Radford, 2003; Noss & Hoyles, 1996; Borba & Villarreal, 2006), referring to the potentiality that a specific artifact has with respect to fostering the education process. On the one hand, researchers explicitly refer to a mediation potential of a given artifact, intending the potential support that such an artifact may offer to the accomplishment of a task. On the other hand, some authors do not explicitly address the issue concerning the relationship between the accomplishment of a task and the mathematical knowledge that is the objective of the teaching–learning process.

Such relationship is differently conceived according to different epistemological perspectives and consequently to different ways of evaluating teaching–learning achievements. The approach that we present starts from an epistemological and cognitive analysis of the use of an artifact in accomplishing a task. On one hand we concentrate on the use of the artifact for accomplishing a specific task, recognizing the construction of knowledge within the solution of the task. On the other hand, we analyze the use of the artifact distinguishing between constructed meanings arising in the individual from his/her use of the artifact in accomplishing the task (*personal meanings*, using a terminology inspired by Leont'ev (1964/1976)), and meanings that an expert recognizes as mathematical (*mathematical meanings*) when observing the student's use of the artifact for accomplishing the task. The construction of knowledge relative to the use of the artifact is thus explicitly connected to helping students become conscious of the personal meanings and linking them to mathematical shared meanings. Therefore, any artifact may offer a valuable *semiotic potential* with respect to particular educational goals (Bartolini Bussi & Mariotti, 2008).

In spite of the difficulty that such identification may present, determining the semiotic potential certainly constitutes a basic element for designing any pedagogical plan centered on the use of a given artifact. A fine-grain analysis can be accomplished outlining the different tasks to be proposed and the corresponding meanings that may emerge from using the artifact, as well the mathematical meanings that may be recognizable as didactical goals. Examples of analysis of the semiotic potential have been developed, for instance, by Bartolini Bussi and Mariotti (2008), for two artifacts, the abacus and the particular dynamic geometry environment Cabri-Géomètre[1] (Laborde & Bellemain, 1995).

3.2 Learning and teaching as the evolution of signs

Accomplishing a task makes meanings emerge, but how might the subject become conscious of such meanings and how might such meanings be explicitly related to mathematics? In other words, in the terminology used above, how may personal meanings arising from the use of a certain artifact for the accomplishment of a task become mathematical meanings for students?

Meanings come to life through representatives of different kinds—words, gestures, drawings …—and even through

[1] During our study we used the software Cabri Géomètre II Plus. In this paper the term "Cabri" refers to this version of the software.

complex hybrids as described, for instance, by the notion of *boundle* (Arzarello, 2006) In the following I will use the term sign in a broad sense, in agreement with the shared claim of considering semiotic systems at large (Radford, 2003; Arzarello, 2006). The use of the term sign is inspired by Pierce's work. We intend to overcome the distinction between signified and signifier, assuming an indissoluble relationship between them. That leads us to revise the common conception that meanings pre-exist to their signifiers and to develop the idea of meaning originating in the intricate interplay of signs (for a thoughtful discussion see for instance (Sfard, 2000, p. 42 and following).

The production of a sign derived from the use of an artifact may be spontaneous or explicitly required by a specific task proposed by the teacher; in any case the main characteristic of these signs is their strong link with the actions accomplished with the artifact. As soon as they emerge and come into existence through their expression via any form of external representation they can be socially shared. The crucial role played by signs in their broader sense is explicitly expressed in the notion of semiotic means of objectification introduced by Radford (2003).

When this semiotic process is triggered in the classroom, both the pupils and the teacher may be involved, assuming a common goal oriented towards mathematics. In this part of the process, the teacher's role becomes crucial: with the educational goal of introducing pupils into a social culture, the teacher is asked to play the role of *cultural mediator*, designing a strategy in order to bridge the individual and the social perspective. In others words, in the social interaction the teacher is asked to promote the evolution of signs referring to personal meanings towards signs referring to mathematical meanings. In doing so the teacher is expected to act both at the cognitive and the meta-cognitive levels, fostering the evolution of personal meanings and guiding pupils to be aware of their mathematical status. This process has an intrinsic complexity that entails various issues related to the establishing of classroom norms (Cobb et al., 1993), and the appropriation of specific speech genre (Hasan, 1992; Bartolini Bussi, 1998): in short, related to entering into the specific culture of a community of which the teacher is a representative.

Exploiting its semiotic potential, the teacher makes the artifact function as a *semiotic mediator*. That means that starting from the functioning of the artifact in the accomplishment of the task and from meanings sprouting from that experience the teacher has to guide students to relate these meanings to mathematics.

In summary, our basic assumption claims that the awareness of the semiotic potential of the artifact allows *the teacher to use the artifact as a* ***tool of semiotic mediation,*** exploiting the possibility of guiding students to connect personal meanings that arise from the use of the artifact and mathematical meanings recognizable by an expert in such use.

> Thus any artifact will be referred to as *tool of semiotic* mediation as long as it is (or it is conceived to be) intentionally used by the teacher to mediate a mathematical content through a designed didactical intervention. (Bartolini Bussi & Mariotti, 2008, p. 754).

This approach does not oppose other educational approaches that, inspired by an instrumental approach (Artigue, 2002), express the didactic aims in terms of instrumental genesis (Trouche, 2004). Beyond and not in contrast with the objective of fostering an instrumental genesis or 'converting tools into mathematical instruments' (Guin & Trouche, 1999), our approach focuses on the learning process related to the use of an artifact through a semiotic lens. Our approach intends to add a semiotic perspective related to the hypothesis on the development of classroom discourse in which pupils and teacher are both actively engaged. From this same perspective we may consider artifacts of various nature and different modes of use, belonging to old or new technologies.

Taking a semiotic perspective means to study the teaching–learning process on the one hand recognizing (and assuming) the central role of signs, either as a product or as a medium, in the construction of knowledge; on the other hand, focusing on the link that it is possible to establish between artifacts and signs and consequently on the potential offered by a particular artifact from an educational perspective.

"[…] the link between artifacts and signs overcomes the pure analogy in their functioning in mediating human action. It rests on the truly recognizable relationship between particular artifacts and particular signs (or system of signs) directly originated by them […]." (Bartolini Bussi & Mariotti, 2008, p. 752).

In other words, we studied the development of semiotic processes related to the use of specific artifacts, focusing on signs and describing teaching and learning through the idea of evolution of signs as it emerges and can be observed in the classroom.

From a didactic point of view, the unfolding and developing of the semiotic potential of a given artifact may become the key goal of the teaching–learning activity that can be achieved through social interaction in the classroom.

Thus the educational intervention may be described as a path from the emergence of signs related to the activities done with the artifact towards the appropriation of mathematical signs.

"[…] thinking and making sense (in society as well as in schools) has to be conceived of as *socio-semiotic process* in which oral and written *texts […]* constantly interact in order to bring about improved texts on the part of the interlocutors or even merge into a revised text as a final product of the whole group." (Carpay & van Oers, 1999, p. 303).

In other words, a path leading to the achievement of educational goals can be recognized and interpreted as the development over time of a semiotic process centered around the use of a specific artifact. Thus, besides activities where students face tasks to be accomplished with the use of the artifact and where the first unfolding of the semiotic potential is expected to occur, specific tasks must be designed to foster the development of the semiotic process described above.

3.3 Semiotic activities

The different activities that can be designed and proposed can be classified according to the different types of students' involvement: at the individual or the social level.

Individual production of signs (e.g. drawing, writing and the like). Students are individually engaged in written productions. For instance, after activities with the artifact, students are asked to produce reports on their experience. Narratives together with commentaries and reflections are expected. They may be asked to write, on their own math notebook, the final shared mathematical formulation of the main conclusions coming from the collective discussion (Cerulli & Mariotti, 2003). This type of activities requires an individual contribution and for its very nature it starts to be detached from the contingency of the situated action. Individual productions of signs may be evoked and shared in collective discussions, and even become objects of discussion.

Collective production of signs (e.g. narratives, mimics, collective production of texts and drawings). As mentioned above, social interaction and specifically collective discussions play a crucial role in the teaching and learning process. When a collective discussion assumes the character of a real *Mathematical Discussion* (Bartolini Bussi, 1998), the most crucial part of the semiotic process on which teaching–learning is based takes place. The whole class is collectively engaged in "mathematical discourse" and the teacher promotes the dialectics between different personal meanings and the mathematical meanings that constitute the educational goal.

The role of the teacher becomes fundamental to fostering the evolution of signs, rooted in the activity with artifacts, into mathematical signs. Such evolution is not expected to be either spontaneous or simple, and for this reason it seems to require a purposeful intervention of the teacher that needs to take into account individual contributions in order to exploit the semiotic potentialities rising from the use of the particular artifact.

One of the directions of our study concerned the description of teacher's interventions in the particular case in which the intention of the teacher with respect to the artifact and the didactic goal is clear. We tried to give a description of the action of the teacher that could be more specific and functional than the generic hint of guiding the evolution of signs. During a number of teaching experiments, our study focused on the analysis of the teacher's intervention with the aim of identifying possible patterns of actions that could be related to the specific intention of fostering the process of semiotic mediation related to the use of a specific artifact. This way, it became possible to outline different categories of actions according to their scope and the circumstances of their occurrence, and a structure consisting of the combination of actions belonging to particular categories. In the following section, I will present this model and provide examples. In order to contextualize such examples, I will provide a short description of one of the teaching experiments on which the study was based.

4 A teaching experiment inspired by the semiotic mediation approach

The model presented above provides both a frame within which teaching–learning sequences can be designed, and a lens through which the educational process can be analyzed.

The notion of semiotic potential is crucial for the design-phase, because during this phase the identification of such potential has to be put in relation with the educational goals set by the teacher, with the actual formulation of the tasks to be proposed to the students, and with the directions the teacher wants the collective discussions to take. The main structure of a teaching intervention may be described as the iteration of what we called didactic cycle (Bartolini Bussi & Mariotti, 2008, p. 754). A didactic cycle consists of a sequence of different activities, each aimed at developing different components of the complex semiotic process described above. A didactic cycle starts with activities that ask students to solve a task using the artifact; it continues with activities that ask students to produce individual signs, and the cycle ends with collective activities in which the teacher orchestrates the evolution of students' personal meanings.

4.1 Tools of a DGE: the semiotic potential

Let us start with the very first step of the utilization of our model in the realization of a teaching experiment: the identification of the semiotic potential of an artifact. We will do it for a very particular artifact: the dynamic geometry environment Cabri-Géomètre. The example we are going to elaborate upon concerns the relationship between some tools of Cabri and their use, and the mathematical notion of function.

On the one hand we consider certain components of Cabri and their use, such as basic points and points obtained through a construction, the dragging tool and its effect on the different kinds of points, the trace tool and the effect of its activation, the macro tool and its functioning with respect to a construction; on the other hand we consider the mathematical notion of function and all the related notions such as that of independent and dependent variables, parameter, domain, image, and finally that of graph. It is possible to identify a rich system of meanings, emerging from the use of the Cabri tools and the corresponding system of meanings related to the mathematical notion of function. We will give a brief account of this related system.

Motion certainly constitutes the main feature of a DGE. Motion is obtained through the use of what is commonly called the dragging tool that is activated through acting with the mouse on different objects on the screen. We will limit ourselves to the case of points even though other kinds of objects can be acted upon through the dragging tool. Points can move in two main ways: according to the direct and the indirect motions.

- The "direct motion" of a point (for instance a basic point) obtained by the direct action on it, represents the variation of this element on the plane. This is the way of representing, in Cabri, a *generic point* on the plane. Consistently, the motion of a *point on an object* represents the variation of a point within a specific geometrical domain, a line, a segment, a circle, and the like, and consequently a generic point belonging to a particular geometrical figure.
- The "indirect motion" of an element occurs when a construction has been accomplished; in this case, the motion of the new elements obtained through the construction is obtained as a consequence of dragging the basic points from which the construction originates; this motion will preserve the geometrical properties defined by the construction. In this way, the indirect motion of a point represents its variation, but such variation depends on the variation of other points through a relation stated by the construction. As a consequence, the use of the dragging tool will allow the user to experience the combination of two interrelated motions, that of basic points and that of constructed points. In other words, the use of the dragging tool may be considered in relation to the idea of function as co-variation between dependent and independent variables.

Further analysis (Laborde & Mariotti 2002; Mariotti, Laborde & Falcade, 2003; Falcade, Laborde & Mariotti 2007) highlights other potentialities of the Cabri environment: other tools can be identified offering a semiotic potential with respect to the notion of function. The macro tool realizes a given construction: whenever applied to the required "initial elements", the macro will produce the corresponding "final elements". The Trace tool displays the trace of a moving point, i.e., its trajectory: it is possible to obtain the trajectory of both independent and dependent variable points. The two correlated trajectories appear progressively, while they are generated point by point, and finally they can be globally perceived as two sets of points. All that can be referred to both the notion of domain and that of image of a function.

4.2 The teaching sequence

A teaching experiment involving Italian and French 10th grade classes was designed and carried out. The classroom experimentations lasted approximately 2 months and were repeated during three academic years. Carrying out the sequence in each classroom took approximately 2 months.

Taking a semiotic mediation perspective, the educational goal was that of introducing students to the idea of function as co-variation using Cabri as a tool of semiotic mediation. The design of the sequence of activities was consistent with the structure of the didactic cycle described above (for a detailed description of the sequence and its re-elaboration during the teaching experiment, see Mariotti, Laborde & Falcade 2003; Falcade, Laborde & Mariotti 2007; Falcade 2006). Different kinds of data were collected: any kind of students' production (worksheets, or written reports), traces of classroom activities were recorded and transcribed. In the following, I will give a general description of the sequence, and a specific account of the first phase from which the illustrative examples are drawn.

4.2.1 General structure of the sequence

The sequence is organized according to three main educational goals:

1. A first formulation of a definition of function is socially constructed in the classroom. The achievement of this goal is based on the interpretation of particular geometric situations in terms of function (as well as image, pre-image, domain, range, co-domain). "Dragging", "Trace tool", and "Macro tool" are the key elements of the artifact that are exploited as tools of semiotic mediation.
2. This phase is focused on a generalization of the definition of function from the geometric context to the numerical context, the introduction of the problem of geometrically representing numerical functions, and

the definition of graph as a geometrical function associated to a numerical one through a well-defined process.
3. Finally, the use of the graph of a function is promoted as a means to solve problems.

4.2.2 *The first part of the sequence*

The examples discussed in this paper concern the first phase of the sequence. Therefore, I will give some more information on the tasks used in this phase.

According to our analysis about the semiotic potential, the first and the second task aim to introduce students to variation and co-variation through exploring the effect of a macro construction. A macro construction is a complex tool that provides a geometrical object as the final product of a construction procedure, when the initial objects are given. As a consequence, a macro construction embeds a functional dependency between initial objects and final objects. In the first and the second tasks, students have to explore the situation produced by two different macros. In the first case, given three free points A, B, P a macro-named Effetto1—provides point H as the orthogonal projection of point P onto line AB.

The exploration is guided by a worksheet, where questions, slightly different in the two tasks, address different aspects. The first question of the first task asks students to explore systematically the effect of the dragging tool on each point appearing on the screen. In a second question, they are asked to observe what happens after the activation of the Trace tool and then to describe the movement of the different points, using the current language of geometry. Figure 1 shows an image of the screen after the activation of the trace tool.

4.3 The role of the teacher

Taking into account the previous discussion and in particular the structure of the didactic cycle, the teacher is expected to intervene at two key moments.

- In the design of the tasks to be accomplished by the students, and subsequently in monitoring the unfolding of the semiotic potential during the activities in the Computer Lab, the teacher's choice is directed by the intention of fostering students' personal production of signs: not only the type of task but also the organization of the classroom activity plays a fundamental role. For instance, asking students to work in pairs at the computer is expected to foster social exchange, accompanied by words, sketches, gestures, and the like. In other words, it is expected to foster the spontaneous production of signs related to the use of the artifact. Moreover, specific tasks can be designed to induce students to activate a semiotic process. Students may be engaged individually in different semiotic activities concerning written productions. For instance, after Lab Activities, students may be asked to write at home individual reports on what happened in the laboratory, adding personal reflections and commentaries. These texts provide a good base for triggering the semiotic process that is envisaged. In particular they provide a first product of a detachment from the action accomplished. They also provide permanent signs—in particular written words—that can be intentionally retrieved in the successive collective activities to be shared by the class community.

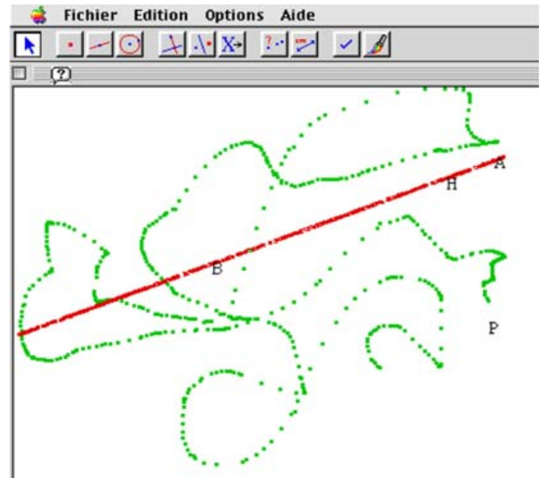

Fig. 1 Traces as they appear on the screen

- The intervention in the classroom discourse. Because of the crucial role that we assume that the teacher plays, collective discussions are specifically planned in the didactic cycle with the goal of organizing the development of semiotic chains leading to the students' appropriation of envisaged mathematical signs. A collective discussion engages the whole class in a collective discourse, which has to become a mathematical discourse.

"Mathematical discussion is conceived of as polyphony of articulated voices on a mathematical object that is one of the motives for the teaching–learning activity […]. (Bartolini Bussi, 1998, p. 68).

The teacher who usually explicitly declares the theme initiates a collective discussion. The occasion may be either the discussion of the solutions to a previous problem-solving activity, or the analysis and re-elaboration of texts produced by the students, or the need to formulate a shared definition of a mathematical idea. The main goal is promoting the "dialectics between different personal senses and the mathematical meaning" (op. cit.). Although the whole class is involved, the role of the teacher is crucial.

However, it is quite difficult to fully explain its nature. According to our framework, the teacher needs to exploit the semiotic potential offered by the artifact, taking into account individual contributions and fostering the move towards mathematical meanings. The ways in which the teacher makes all this happen were the focus of our study. In this contribution, we will present a model of the role of the teacher in the process of semiotic mediation. The following examples aim at illustrating this model. Other partial analyses of the teacher's intervention can be found in (Bartolini Bussi 1998, Mariotti & Bartolini Bussi 1998; Mariotti, 2001; Bartolini Bussi, Mariotti & Ferri, 2005; Cerulli, 2004).

4.4 The mediation of the teacher/the teacher as mediator: a recurrent pattern of intervention

As illustrated above, a teaching experiment was carried out, based on the implementation of the teaching sequence we described. The collective discussions that took place in the classroom were recorded and the transcripts analyzed with the aim of describing and finally explaining the role of the teacher in the development of the mediation process. What we were looking for was to identify teacher's interventions that could be referred to the intentional utilization of the artifact as a tool of semiotic mediation. In other words, we were interested in identifying specific semiotic games (Mariotti & Bartolini Bussi, 1998; Arzarello & Paola, 2007) played by the teacher when intervening in the discourse to make the students' personal senses emerge from the common experience with the artifact, and develop towards shared meanings, consistent with the mathematical meanings that are the object of the educational project. The analysis of the transcripts highlighted a recurrent pattern, i.e., a recurrent sequence of interventions, where it is possible to recognize the intention of the teacher to exploit the semiotic potential of the artifact. The pattern is constituted by four categories of interventions that can be grouped in two complementary pairs.

The first pair collects categories of intervention that share the common goal of promoting both the unfolding of the semiotic potential of the artifact and the co-construction of common signs. More specifically, the goal consists in fostering the individual production of signs related to the use of the artifact, meanwhile securing that students share the context of reference and some of its key elements.

We named the first two categories *Ask to go back to the task* and *Focalize on certain aspects of the use of the artifact*.

The second pair of categories, named, respectively, *Ask for a synthesis* and *Provide a synthesis*, share the goal of making signs overcome the point of view of the individual to acquire the needed generality, and at the same time the goal of taking the point of view of the community of the mathematicians.

In the following section, we will give a short description of these categories. The discussion of few examples will accompany and illustrate such description.

4.4.1 Co-construction of shared signs

Ask to go back to the task. This category collects those types of teacher's interventions aiming to reconstruct the context of the task and in particular the modality of use of the artifact in the solution of the task. A typical intervention is that of asking to recall the question posed by the task and how the artifact was used in that circumstance. This can be considered a request of making explicit the utilization schemes mobilized in the solution of the tasks. In other words, when students are asked to reconstruct their experience with the artifact, their answer may be the case of an actual reconstruction as well as that of a recount based on one's own memory. The objective is that of making signs emerge in relation to the experience with the artifact under the stimulus of the reconstructed context. Actually, this kind of intervention usually results in the production (or re-emergence) of signs strictly related to the use of the artifact. This is fundamental to starting (or re-starting) the development of new meanings, since a social endeavor asks for a shared base on which to start the evolution.

Excerpt 1 First discussion

(12) Teacher: Well, then…let's see if looking back at what we have done, we can find what we want our idea of function to be…so…what have you done, tell me so I'll do it too [the teacher is ready to act on the computer]…who can tell me?

(13) BA: I'll tell you…so…we drew points A, B, and P, anywhere and then we applied the macro construction effect to points A, B and P in this order and we got another point which we called H. [in the mean time, the teacher does the construction on the computer and the image is projected for the whole class].

This is a very typical start for a collective discussion. The recount is accompanied by its realization on the computer and projected on a screen. This is not always the case; sometimes the discussion takes place in a room where no computers are available.

The teacher invites the students to go back to what they did and explicitly declares that the final didactic goal is to develop a common idea of function. The fact that this idea has to be shared with the teacher gives the students an implicit message, as far as the teacher is recognized as an expert and a representative of the mathematicians' community, the students know that what they are going to elaborate must be consistent with the mathematical notion of function.

Excerpt 2 Second discussion

(1) Teacher: What we stopped on the other day, remember, was the problem of what would happen when doing our construction over again…the point H could disappear or not. Each of you, on your papers, has told me what you thought and that is what I would like to share again now.

This second excerpt shows another example of a "back to the task" intervention. In this case, the teacher explicitly refers to the written reports produced by the students. In so doing, the teacher aims to foster the re-emergence of signs related to the artifact from the memory of both the Lab Activities and the reflective activity of writing the report. The construction of a shared context is fundamental in order to reach shared meanings expressed by shared signs. The re-construction of a common context related to the experience with the artifact is the base on which the common signs emerge. Such signs still have a link with the use of the artifact, but they also have the potentiality to evolve towards mathematical signs; because of their link with the artifact we named them "artifact signs" (Bartolini Bussi & Mariotti, 2008, p. 751). Consider the first task and the experiences related to activating the macro Effetto1, dragging the different points and observing what happens on the screen and the different behaviors of the points. After the first discussion a number of expressions, such as "moving point/s" or "fixed (still) point/s", assume complex meanings that overcome the obvious reference to the dynamic phenomena produced on the screen to include the reference to the relation of dependency linking them.

The construction of such a complex net of meanings related to a specific expression is achieved by a recurrent use of "back to the task" interventions that are cyclically repeated and used whenever the teacher feels the need to recover the experience lived in the context of the artifact. Consider, for instance, the following intervention that occurs after that of excerpt 1, and that aims to re-direct the discussion towards the request of the task.

Excerpt 3 First discussion

(21) Teacher: Yes, because now you were led to discover this construction…why? what was said? I mean, what were you asked to do?

(22) BA: We had to say…first if we moved point A … which were the points that moved and didn't move…

These kinds of teacher's intervention are only partially planned in advance; they are mostly produced as *on the spot* reactions to students' behavior. Sometimes these interventions may appear redundant, but their recurrence also aims to enlarge the participations as much as possible, in order to achieve a collaborative construction of meanings. Of course the mobilization of a large number of contributions may leave space for the appearance of a lot of spurious elements. For this very reason the teacher utilizes a second, complementary type of intervention with the main objective of focusing on the specific elements of the shared context that are of interest. The teacher intentionally selects some of the emerging artifact signs and attempts to limit their semantic field; in other words, her intervention consists in *Focalizing on certain aspects of the use of the artifact*. Consider, for instance, the following part of the previous excerpt 1 drawn from the 1st Collective Discussion (the complete excerpt is reported for the reader's convenience). After the recount provided by BA (13) the teacher intervenes: her utterance (14) opens with a request of attention "stop here … here, there is something …", and is followed by an intervention where students' attention is directed towards the macro. We classify this intervention as *focalization*. This *Focalization* category collects all teacher interventions where in a more or less explicit way students' attention is directed on particular aspects of their experience (past or present). In these occasions, gestures or changes in the tone of the voice are often observed showing the intentionality of focalizing. This type of interventions can be considered complementary to the previous ones. In fact, following a back to the task intervention, a focalization highlights the use of certain signs, selecting pertinent aspects of their shared meanings in respect to the development of the mathematical signs that constitute the final education goal. Consider the following excerpt that includes the previous one and shows how the "back to the task" intervention is followed by a "focalization" intervention.

Excerpt 4 First discussion

(12) Teacher: Well, then…let's see if looking back at what we have done, we can find what we want our idea of function to be…so…what have you done, tell me so I'll do it too…who can tell me?

(13) BA: I'll tell you…so…we drew points A, B, and P, anywhere and then we applied the macro construction to points A, B and P in this order and we got another point which we called H.

(14) Teacher: Ok, let's stop here…there is something…I mean if I had to see this effect 1…what do you think the macro effect 1 is?

(15) BA: I mean, it's the construction that…there is a hidden construction behind it that allows us to…draw point H starting from points A, B, and P.

(16) Teacher: Effect 1 condenses, hiding it, a construction that you then discovered…and what does this construction do?

(17) BA: It constructs a point, it constructs point H…because we did…
(18) Teacher: It constructs point H starting from?
(19) Chorus: The three points.

As shown in the excerpt above, the interventions of these two categories appear to be interlaced: a back-to-task action (12) is followed by a focalization (14). From the recalling of the task the teacher decides to select a relevant aspect and she focuses on the macro, asking an interpretation of that macro. The student explains the macro in terms of the hidden construction—"there is a hidden construction behind it" (15)—and in terms of the characteristics of a macro, that are the initial elements, the final elements and their relationship of dependence—"that allows us to…draw point H starting from points A, B, and P". The next lines of the excerpt repeat this explanation process, making the key elements—"It constructs point H from the three points"—clear for everyone.

Immediately after the teacher (excerpt 5 below) shifts the focus from the general to the particular and asks the students to recall the dragging experiences and the dependence of movement of a point from the movement of another.

Excerpt 5 First discussion
(21) Teacher: Yes, because now you were led to discover this construction…why? what was said? I mean, what "did you have to do"?
(22) BA: We had to say…first if we moved point A which were the points that moved and didn't move…
(23) Teacher: Ok, then…for example, moving P, I see that only H moves and not only,…I also see that what moves…?

This example clearly shows that the teacher's interventions are intentionally directed to support students to become conscious of the key aspects of their experience. Although everybody had the same experience, it will be through making it explicit that students will become aware of the fact that particular elements can be selected and isolated from the multiplicity of sensations.

Verbalization plays a key role in this process of gaining consciousness. For this reason, the teacher iterates her request of describing the experience in order to make certain words *crystallize* (Moreno, Hegedus, Kaput, 2008) the experience for all the students.

In the following Excerpt 6, we can observe how the coordination of different observed movements, i.e., an instance of co-variation, is repeatedly expressed and focalized, after the intervention of the teacher.

Excerpt 6 First discussion
(31) Teacher: What is point H!? … wait, let's hear someone else … TA … come on! … then what can I do?
(32) TA: you can move the other points
(33) Teacher: you can move the other points … so, for example … should I move A?
(34) BA: Yes … on the circle …
(35) Teacher: So, some people had trouble with this at the beginning, but anyway you can see that H … where is it [placed]?

As discussed above, the two types of operations (back to the task and focalization) are complementary. The movement from one type of intervention to the other is not one-way oriented; on the contrary, we can observe a double movement that is accomplished also through the use of other types of interventions.

4.4.2 Towards mathematical signs

As could be easily foreseen, both in the written reports of the students and in the utterances of first collective discussions, it is very common to observe expressions like "it moves", and "it does not move", "moving point", "point on an object", "macro", and the related "initial objects", "final objects". Their meanings become shared and stable: they are rooted in the common experience with the artifact and condense the key elements that emerged through the focalization process triggered by the teacher. In other words, their use witnesses a consolidation of signs that are directly related to the use of the artifact (we named them "artifact signs", Bartolini Bussi and Mariotti, 2008 p. 756[2]), and belongs to the first stage from which the evolution towards the mathematical signs is expected to start. The movement towards the elaboration of mathematical signs requires the detachment from the artifact that cannot be limited to a change in the signifier. For instance, it is not sufficient to re-name a "moving point" as "independent variable" to assure that this new expression has gained the full mathematical meaning. At the same time, through their evolution, meanings should maintain some of the crucial aspects coming from their origin. For instance, the dynamic component rooted in the experience of moving points should remain as part of the meaning of the sign < independent variable > whatever mathematical definition of function will be finally formulated.

All this requires a complex semiotic process that needs time and purposeful interventions to be developed. The teacher is not only a co-actor, he/she is one of the key movers of this process, often acting as a catalyst. The

[2] A classification of signs is fully described in this reference, where it is also explained how the appearance of signs belonging to different categories may be used to describe the evolution of the semiotic process. (op. cit. pp. 765–58).

second pair of categories of intervention that we are going to outline in the following section intends to describe some aspects of the teacher's role as a mover towards the emergence of mathematical signs. As illustrated above, they are named respectively, *Ask for a synthesis* and *Provide a synthesis*, and have the common goal of fostering the movement from the perspective of the individual towards a de-contextualized and generalized perspective, that should be consistent with the point of view of the community of the mathematicians.

Ask for a synthesis. This category concerns all the operations aimed at soliciting the students to synthesize, that is to condense in a few sentences what has been done and discussed in the classroom up to a certain moment. This request is commonly interpreted by the students as the request of making explicit what they have understood. This request of synthesizing aims not only at inducing students at the same time to make explicit personal meanings but also to take into account the first results of sharing meanings in social interaction. Synthesizing is expected—though not certain—to induce students to generalize, and the intervention can be considered successful when a process of generalization is triggered.

It may happen that the synthesis produced by a student refers to exchanges that have occurred during the current or the previous collective discussions, and that it involves expressions previously emerged (including mathematical expressions used by the teacher). Consider the following excerpt, drawn form the last part of the first collective discussion. The teacher asks the students to synthesize, trying to also involve someone who did not intervene before.

Excerpt 7 First discussion

(211) Teacher: Who would like to synthesize all what I have said? … but I want someone that never talked … MA!

(212) MA: what I understood…?

(213) Teacher: Ok, go on, what did you understand

(214) MA: I mean … there are certain things that are taken from others that are independent… that are points A, B and P; H is obtained by a construction that derives from A, B and P, thus H depends on the position …

(215) Teacher: … on the position of the three points A, B and P. Thus the function … what is it [the function] for you?

(216) MA: The function for me is … I mean it should be a construction that practically … is obtained by different means … that derive from …

(217) Teacher: From which points?

(218) MA: A, B, and P.

(219) Teacher: OK.

The teacher mirrors MA's question and explicitly asks "what did you understand?". As expected, the beginning of a de-contextualization process appears. MA's utterance contains generic terms—"certain things taken from others". Although they are general, such terms maintain their reference to their origin and that allows MA to turn back to speak of moving points in the geometrical construction (artifact signs). After following the pupils in the explication within the artifact context, the teacher comes back to the start, asking to make explicit the personal meaning of the sign < function > and MA restarts from a more general point of view.

This short excerpt shows how the evolution may progress: back and forth from the artifact context to the mathematical context. Expressions as "certain things" or "depends on", seem to play a hinge-role, connecting the two contexts, but also fostering the movement from one to the other. For their specific hinge-role we have named these signs *pivot signs* (Bartolini Bussi & Mariotti, 2008, p. 757). Moreover, we can observe that slowly, but continuously, the teacher pushes the students to abandon the reference to the artifact context, selecting specific qualities from the use of the artifact to be transferred to the mathematical context.

Generally speaking, these interventions aimed at making students synthesize are expected to contribute to the development of the interpersonal space (Cummins, 1996), within which mathematical signs might be produced and put in relation with the artifact signs. The personal meanings are shared through students' syntheses and form the shared semiotic environment within which the teacher may introduce the point of view of mathematics, and eventually a standard terminology. The process of evolution of signs has to develop from the consolidation of artifact signs towards the introduction of mathematical signs. The fourth category of intervention has an important role in this development.

Provide a synthesis. This category collects the interventions of the teacher aimed to retrieve particular signs and to fix their use in the classroom discourse and more specifically fix them with respect to mathematics. The objective of these interventions is to explicitly ratify the acceptance of a sign, the use and status of which are related to the mathematical context. These interventions aim to summarize and highlight semiotic relationships between signs that are already shared in the class community. The teacher intends to produce stable semiotic links. Thus, the success of this kind of intervention constitutes a fundamental step in the development of the semiotic mediation process. In the following Excerpt, we have an example of an intervention that can be classified as a case of Provide a synthesis. At (159) the teacher explains the relationship between independent and dependent variables in a function, showing once more a great care in evoking the artifact context.

Excerpt 8 First discussion

(159) Teacher: Well, then what happens is that in general for a function, the points from which I start are named independent variables, because I can move them wherever I like, whilst what I obtain is named dependent variable, because it depends … on what [does it depend]?

(160) MO: [it depends on] the independent variables.

This intervention is an exemplar. The teacher fixes the use of the mathematical terms "independent variable" and "dependent variable" making explicit how their meaning is related to the artifact and in particular to certain artifact signs, <point from which I start>, <points I can move wherever I like>. At the same time, the teacher refers explicitly to the generality of the use of these terms and in this way opens to the need of overcoming the limits of the context of the artifact and of moving into the mathematics domain.

The alternation of interventions belonging to this second pair of categories is aimed at directly involving students in generalization and de-contextualization processes. At the same time, it is aimed at giving them the possibility of appropriating of the mathematical signs that are introduced by the teacher and linked to the new meanings emerging form the collective discussion.

5 Conclusions

The pedagogical model, based on the construct of semiotic mediation and presented in the first part of this paper, foresees a key role for the teacher. Our study focused on analyzing this role with the objective of elaborating a description that could shed light onto the general functioning of the mediation process in the teaching–learning activity. Moreover, the model presented above provides a general frame within which to describe the teacher's purposeful interventions aimed at fostering the process of semiotic mediation centered on the use of a particular artifact. Through the use of the different categories described it is possible to analyze the teacher's role in the evolution from personal meanings to mathematical meanings.

The use of interventions of the first pair of categories can be put in relation with the unfolding of the expected semiotic potential of a given artifact. The first pair of categories helps to identify the conditions that seem necessary to move from the experience with the artifact towards the consciousness of its relevant and pertinent aspects with respect to the mathematical meanings that constitute the educational goal.

Interventions from the categories *Ask to go back to the task* and *Focalize on certain aspects of the use of the artifact* can contribute to describe that part of the internalization process "concerned with how consciousness emerges out of human social life" (Wertsch & Stone, 1995, p. 164). At the same time, these categories allow to describe how this move is triggered by the teacher and accomplished within collective discourse.

The use of interventions from the second pair of categories can be put in relation with the achievement of the educational goals. The second pair of categories helps to identify how it is possible to trigger the process of detachment from the artifact and the emergence of signs that at the same time reach a certain generality and a mathematical status. The model outlined above contributes to shed light onto the delicate but crucial role that teacher has as cultural mediator.

As Siemon et al. claimed, current interactionist perspectives "point to the need for a deeper understanding of the ways in which teachers contribute to the shaping of classroom cultures" (2004, p. 193).

All the categories of intervention allow us to describe the progression along the educational path according to the original educational project started with the activities with the artifact and based on the assumption of its semiotic potential. Actually, our analysis of the collected data shows the instability of this system and the necessity of reaching equilibrium between the demand of not losing track of maintaining the didactic goals and the need of taking into account what happens in the classroom. A failure in each of these directions would lead the educational project to fail. "Back to the task" interventions may be successful in restoring the link with the artifact when it is lost, while "ask to synthesize" interventions may help to overcome the reference to the real action in order to reach new general meanings.

The evolution from personal meanings to mathematical meanings requires the development of a specific didactic contract (Brousseau, 1997) related to the recognition of the relationship between the experience with the artifact and mathematical knowledge. This requires teacher's interventions aimed at shifting the discourse to a meta-level where a specific contract can be established. It seems that this type of shift can described and modeled by the articulation between interventions belonging to different categories. Specifically, the interventions of the type "provide synthesis" seem to contribute to realize the passage from social norms to mathematical norms, expressed by Cobb, Wood and Yackel (1993). The class community states what is shared, but it is the teacher's responsibility to introduce specific terms and criteria for recognizing what can be referred to as mathematics and how it can be referred to.

Further research aimed at the refinement of this model is in progress. A highly promising direction of investigation concerns the first pair of categories and in particular the identification of specific semiotic games that may nurture the evolution of artifact signs, for instance, exploiting the interplay between different semiotic registers. Similarly, the second pair of categories might be further elaborated with the aim of describing how the teacher's interventions may be shaped to foster the evolution of the specific didactic contract concerning the relationship between personal meanings and mathematical knowledge.

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References

Artigue, M. (2002). Learning mathematics in a CAS environment: The genesis of a reflection about instrumentation and the dialectics between technical and conceptual work. *International Journal of Computers for Mathematical Learning, 7*, 245–274.

Arzarello, F. (2006). Semiosis as a multimodal process. *Relime* Vol Especial, 267–299.

Arzarello, F., & Bartolini Bussi, M. G. (1998). Italian trends in research in mathematics education: A national case study in the international perspective. In J. Kilpatrick, & A. Sierpinska (Eds.), Mathematics education as a research domain: A search for identity (Vol. 2, pp. 243–262). Dordrecht: Kluwer.

Arzarello, F., & Paola, D. (2007). Semiotic games: The role of the teacher. In J. H. Woo, H. C. Lew, K. S. Park, & D. Y. Seo (Eds.), Proceedings of the 31st conference of the international group for the psychology of mathematics education (Vol. 2, pp. 17–24). Seoul, South Korea.

Bartolini Bussi, M. G. (1998). Verbal interaction in mathematics classroom: A Vygotskian analysis. In H. Steinbring, M. G. Bartolini Bussi, & A. Sierpinska (Eds.), *Language and communication in mathematics classroom* (pp. 65–84). Reston: NCTM.

Bartolini Bussi, M. G., & Mariotti, M. A. (2008). Semiotic mediation in the mathematics classroom: Artifacts and signs after a Vygotskian perspective. In L. English, M. Bartolini Bussi, G. Jones, R. Lesh, & D. Tirosh (Eds.), *Handbook of international research in mathematics education, second revised edition*. Mahwah: Lawrence Erlbaum.

Bartolini Bussi, M. G., Mariotti, M. A., & Ferri, F. (2005). Semiotic mediation in the primary school: Dürer glass. In M. H. G. Hoffmann, J. Lenhard, & F. Seeger (Eds.), *Activity and sign—grounding mathematics education: Festschrift for Michael Otte* (pp. 77–90). New York: Springer.

Bauersfeld, H. (1980). Hidden dimensions in the so-called reality of the mathematics classroom. *Educational Studies in Mathematics, 11*(1), 23–41.

Borba, M. C., & Villarreal, M. E. (2005). *Humans-with-media and the reorganization of mathematical thinking: Information and communication technologies, modeling, visualization and experimentation*. New York: Springer.

Brousseau, G. (1997). *Theory of didactical situations in mathematics*. Dordrecht: Kluwer.

Carpay, J., & van Oers, B. (1999). Didactical models. In Y. Engeström, R. Miettinen, & R. Punamäki (Eds.), Perspectives on activity theory. Cambridge: Cambridge University Press.

Cerulli, M. (2004). Introducing pupils to algebra as a theory: L'Algebrista as an instrument of semiotic mediation, Ph.D Thesis in Mathematics, Università di Pisa, Scuola di Dottorato.

Cerulli, M., & Mariotti, M. A. (2003) Building theories: Working in a microworld and writing the mathematical notebook. In N. I. Pateman, B. J. Dougherty, & J. Zillox (Eds.), Proceedings of the 27th PME (Vol. 2, pp. 181–188). Honolulu: Hawaii.

Cobb, P., Wood, T., & Yackel, E. (1993). Discourse, mathematical thinking and classroom practice. In E. A. Forman, N. Minick, & C. A. Stone (Eds.), *Contexts for learning: Sociocultural dynamics in children's development*. New York: Oxford University Press.

Cummins, J. (1996). *Negotiating identities: Education for empowerment in a diverse society*. Ontario: California Association of Bilingual Education.

Da Ponte, J. P., & Chapman, O. (2006) Mathematics teachers' knowledge and practices. In A. Gutierrez, & P. Boero (Eds.), Hand book of research on the psychology of mathematics education (pp. 461–512). Rotterdam: Sense Publisher.

Di Sessa, A. A., & Cobb, P. (2004). Ontological innovation and the role of theory in design experiments. *Journal of the Learning Sciences, 13*(1), 77–103.

Drijvers, P., Kieran, C., & Mariotti, M. A. (2009). Integrating technology into mathematics education: theoretical perspectives. In C. Hoyles, J. -B. Lagrange (Eds.), Digital technologies and mathematics teaching and learning: Rethinking the terrain. New York: Springer (in press).

Falcade, R. (2006). Théorie des Situations, médiation sémiotique et discussions collective, dans des sequences d'ensignament avec Cabri- Géomètre por la construction des notions de fonction et graphe de fonction. Grenoble: Université J. Fourier: Unpublished doctoral dissertation.

Falcade, R., Laborde, C., & Mariotti, M. A. (2007). Approaching functions: Cabri tools as instruments of semiotic mediation. *Educational Studies in Mathematics*, *66*(3), Dordrecht: Kluwer, pp. 317–333.

Gravemeijer, K. (1994). *Developing realistic mathematics education*. Utrecht: Cdß Press.

Gravemeijer, K. (1998). Developmental research as a research method. In J. Kilpatrick & A. Sierpinska (Eds.), *Mathematics education as a research domain: A search for identity* (pp. 277–295). Dordrecht: Kluwer.

Guin, D., & Trouche, L. (1999). The complex process of converting tools into mathematical instruments: The case of calculators. *International Journal of Computer for Mathematical Learning, 3*, 195–227.

Hasan, R. (1992). Speech genre, semiotic mediation and the development of higher mental functions. *Language Science, 14*(4), 489–528.

Laborde, J.-M., & Bellemain, F. (1995). *Cabri-géomètre II and Cabri-géomète II plus [computer program]. Dallas, USA: Texas instruments and Grenoble*. France: Cabrilog.

Laborde, C., & Mariotti, M. A. (2002) Grounding the notion of function and graph in DGS, *Actes de CabriWorld 2001—*Montreal.

Lave, J., & Wenger, E. (1991). Situated learning: Legitimate peripheral participation. Cambridge: Cambridge University Press.

Leder, G., Pehkonen, E., & Törner, G. (Eds.). (2002). *Beliefs: A hidden variable in mathematics*? Dordrecht: Kluwer.

Leont'ev, A. N. (1976 orig. Ed. 1964). *Problemi dello sviluppo psichico*. Editori Riuniti and Mir.

Mariotti, M. A. (2001) Justifying and proving in the cabri environment. *International Journal of Computer for Mathematical Learning, 6*(3), Dordrecht: Kluwer, 257–281.

Mariotti, M. A., Laborde, C., & Falcade, R. (2003). Function and graph in DGS environment, Proceedings of the 27 PME conference, Hawaii, III- 112-120.

Meira, L. (1998). Making sense of instructional devices: the emergence of transparency in mathematical activity. *Journal for Research in Mathematics Education, 29*(2), 121–142.

Moreno, L. M., Hegedus, S. J., & Kaput, J. J. (2008) From static to dynamic mathematics: historical and representational perspectives. *Educational Studies in Mathematics, 68*(2), Springer, 99–111.

Noss, R., & Hoyles, C. (1996). *Windows on mathematical meanings*. Dordrecht: Kluwer.

Radford, L. (2003). Gestures, speech, and the sprouting of signs: A semiotic-cultural approach to students' types of generalization. *Mathematical Thinking and Learning, 5*(1), 37–70.

Sfard, A. (2000) Symbolizing mathematical reality into being—or how mathematical discourse and mathematical objects create each other. In P. Bobb, E. Yackel, & K. McClain (Eds.), Symbolizing and communicating in mathematics classrooms (pp. 37–98), Laurence Erlbaum

Siemon, D., Virgonna, J., Lasso, M., Parsons, V., & Cathcart, J. (2004). Elaborating the teacher's role—towards a professional language. Proceedings of the 28th conference of the international group for the psychology of mathematics education (Vol. 4, pp. 193–200). Norway: Bergen.

Thompson, A. G. (1992). Teachers' beliefs and conceptions: A synthesis of the research. In D. Grouws (Ed.), *Handbook of research in mathematics teaching and learning* (pp. 127–146). New York: Macmillan.

Trouche, L. (2004). Managing the complexity of human/machine interactions in computerized learning environments: Guiding students' command process through instrumental orchestrations. *International Journal of Computers for Mathematical Learning, 9*, 281–307.

Van den Akker, J., Gravemeijer, K., McKenney, S., & Nieveen, N. (Eds.). (2006). *Educational design research*. London: Routledge.

Vygotsky, L. S. (1978). Mind in society: The development of higher psychological processes. Harvard University Press.

Vygotsky, L. S. (1981). The genesis of higher mental functions. In J. V. Wertsch (Ed.), *The concept of activity in Soviet psychology*. Armonk: Sharpe.

Wertsch, J. V., & Addison Stone, C. (1985). The concept of internalization in Vygotsky's account of the genesis of higher mental functions. In J. V. Wertsch (Ed.), *Culture, communication and cognition: Vygotskian perspectives* (pp. 162–166). New York: Cambridge University Press.